Homework 4

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Problem 1. *Let p be a proposition. Let Pr be a probability function such that $Pr(p) = .6$. What is $Pr(\neg p|p)$?*

Problem 2. *On any given day, the probability that it rains and there are clouds in the sky is* $.3$. *And on any given day, the probability that there are clouds in the sky is* $.6$. *What is the probability of it raining today given that today, there are clouds in the sky?*

Problem 3. *Scientists run an experiment to determine whether or not electrons are negatively charged. They isolate an electron, and prepare to use it to test their theory. According to that theory, electrons are indeed negatively charged: so the scientists' credence in the electron being negatively charged, given the theory, is* 1. *Before doing the experiment, the scientists have* $.5$ *degree of confidence that this electron is negatively charged. And before doing the experiment, the scientists have* $.2$ *degree of confidence in their theory. What ought the scientists' degree of confidence in their theory be, once they run their experiment and they observe that, indeed, the electron's charge is negative?*

Problem 4. *Two factories manufacture blenders.* 3% *of the blenders produced by factory A are defective: in other words, if a blender is randomly chosen from all the blenders produced by factory A, the chance of that chosen blender being defective is* $.03$. 5% *of the blenders produced by factory B are defective: in other words, if a blender is randomly chosen from all the blenders produced by factory B, the chance of that chosen blender being defective is* $.05$. *Suppose that* 40% *of the world's blenders are produced by factory A: in other words, if a blender is randomly chosen from all the blenders in the world, the chance of that blender having been produced at factory A is* $.4$. *The rest of the world's blenders are produced by factory B. And of course, no blender is produced by both factories.*

With all that as background, answer the following question: what is the chance that a randomly chosen blender will be defective?

Problem 5. *Assume probabilism: that is, assume that rational agents' credences ought to conform to the probability axioms. Suppose that the stochastic truth table below represents Michael's credences in each of four possible states of the world.*

State	p	q	$Pr(w_i)$
w_1	*T*	*T*	*.2*
w_2	*T*	*F*	*.3*
w_3	*F*	*T*	*.45*
w_4	*F*	*F*	*.05*

Use this table to answer the following questions.

1. *After learning p, what should Michael's credence in q be?*
2. *After learning $p \vee q$, what should Michael's credence in q be?*
3. *After learning $\neg(p \leftrightarrow q)$, what should Michael's credence in $\neg p$ be?*

Problem 6. *Assume probabilism: that is, assume that rational agents' credences ought to conform to the probability axioms. Suppose that the stochastic truth table below represents Joseph's credences in each of eight possible states of the world.*

State	p	q	r	$Pr(w_i)$
w_1	*T*	*T*	*T*	*.25*
w_2	*T*	*T*	*F*	*.1*
w_3	*T*	*F*	*T*	*.1*
w_4	*T*	*F*	*F*	*.15*
w_5	*F*	*T*	*T*	*.05*
w_6	*F*	*T*	*F*	*.1*
w_7	*F*	*F*	*T*	*.2*
w_8	*F*	*F*	*F*	*.05*

Use this table to answer the following questions.

1. *After learning p, what should Joseph's credence in r be?*
2. *After learning $\neg q$, what should Joseph's credence in $r \wedge p$ be?*
3. *After learning $\neg(p \vee q)$, what should Joseph's credence in $r \vee p$ be?*

Problem 7. *Let p be a proposition and let Pr be a probability function. Suppose $Pr(p) = .25$. Let $u_p = 3$ be the value—the 'utility'—associated with p being true. Let $u_{\neg p} = 2$ be the value—the 'utility'—associated with $\neg p$ being true. Let X be a random event whose outcome is either p or $\neg p$. What is the expected value of X?*

Problem 8. *Suppose a friend offers you the following bet. An unfair coin will be flipped. The chance of this coin landing heads is* .6, *and of course, the coin always lands either heads or tails (and it never lands both, obviously). Your friend offers to pay you* $5 *if the coin lands tails. If the coin lands heads, however, you must pay you friend* $4.

Assume the following principle of rationality: you ought to take your friends bet if and only if doing so has positive expected value. Given that principle, ought you to take your friend's bet? Solve this problem by calculating the relevant expectation value.